Eur. Phys. J. C 16, 453–460 (2000)
Digital Object Identifier (DOI) 10.1007/s100520000422

THE EUROPEAN
PHYSICAL JOURNAL C

Trimuon production in νN-scattering as a probe of massive neutrinos

M. Flanz[1,a], W. Rodejohann[1,b], K. Zuber[2,c]

[1] Lehrstuhl für Theoretische Physik III, Universität Dortmund, Otto-Hahn Strasse 4, 44221 Dortmund, Germany
[2] Lehrstuhl für Experimentelle Physik IV, Universität Dortmund, Otto-Hahn Strasse 4, 44221 Dortmund, Germany

Received: 16 February 2000 / Revised version: 11 April 2000 /
Published online: 6 July 2000 – © Springer-Verlag 2000

Abstract. The lepton–number violating process $\nu_\mu N \to \mu^- \mu^+ \mu^+ X$ is studied for the first time in connection with Majorana neutrino masses of the second generation. The sensitivity for light and heavy Majorana neutrinos is investigated. The ratio with respect to the standard model charged current process is improved by some orders of magnitude if compared to previously discussed Majorana induced $\Delta L_\mu = 2$ processes. Non–observation of this process in previous experiments allows to demand the effective mass to be $\langle m_{\mu\mu} \rangle \lesssim 10^4$ GeV, being more stringent than previously discussed direct bounds, however still unnaturally high. Therefore, in the forseeable future, indirect bounds on effective masses other than $\langle m_{ee} \rangle$ will be more stringent.

1 Introduction

Investigation of lepton–number violating processes is one of the most promising ways of probing physics beyond the standard model. A particular aspect of this topic is lepton–number violation in the neutrino sector, which in the case of massive neutrinos would allow a variety of new phenomena [1]. This emerges immediately in case of Majorana masses of the neutrinos, which are predicted in most GUT–theories [2]. For ν_e the searches for Majorana neutrinos mainly rely on neutrinoless double beta decay($0\nu\beta\beta$), resulting in an upper limit on the effective Majorana mass $\langle m_{ee} \rangle = \mid \sum U_{em}^2 m_m \eta_m^{\mathrm{CP}} \mid$ of about 0.2 eV [3], where m_m are the mass eigenvalues, $\eta_m^{\mathrm{CP}} = \pm 1$ the relative CP–phases and U_{em} the mixing matrix elements. In general, there is a 3×3 matrix of effective Majorana masses, the elements being

$$\begin{aligned} \langle m_{\alpha\beta} \rangle &= |(U \,\mathrm{diag}(m_1 \eta_1^{\mathrm{CP}}, m_2 \eta_2^{\mathrm{CP}}, m_3 \eta_3^{\mathrm{CP}}) U^{\mathrm{T}})_{\alpha\beta}| \\ &= \left| \sum m_m \eta_m^{\mathrm{CP}} U_{\alpha m} U_{\beta m} \right| \text{with } \alpha, \beta = e,\, \mu,\, \tau. \quad (1) \end{aligned}$$

In this paper we explore the possibility to learn about Majorana masses associated with the second generation. The process under study is muon lepton–number violating ($\Delta L_\mu = 2$) trimuon production in neutrino–nucleon scattering via charged current reactions (CC)

$$\nu_\mu N \to \mu^- \mu^+ \mu^+ X. \quad (2)$$

The muonic analogy for the quantity measured in neutrinoless double beta decay reads $\langle m_{\mu\mu} \rangle = \mid \sum U_{\mu m}^2 m_m \eta_m^{\mathrm{CP}} \mid$ and is investigated for both light and heavy Majorana neutrinos. The relevant diagram is shown in Fig. 1, which also defines the kinematics. Alternative ways discussed in the literature to obtain direct information about $\langle m_{\mu\mu} \rangle$ are muon capture on nuclei [4] and lepton number violating K–decays like $K^- \to \pi^+ \mu^- \mu^-$ [5–8]. The experimental knowledge of effective Majorana masses other than the one measured in $0\nu\beta\beta$ allows only rather poor limits. The best values obtained are from muon–positron conversion in sulfur (therefore sensitive to $\langle m_{\mu e} \rangle^2$) and lepton–number violating K–decays:

$$\begin{aligned} &\frac{\sigma(^{32}\mathrm{S} + \mu^- \to ^{32}\mathrm{Si}^* + e^+)}{\sigma(^{32}\mathrm{S} + \mu^- \to ^{32}\mathrm{P}^* + \nu_\mu)} < 9 \cdot 10^{-10} \\ &\quad \Rightarrow \langle m_{\mu e} \rangle < \begin{cases} 0.4 \text{ GeV (singlet)} \\ 1.9 \text{ GeV (triplet)} \end{cases} \\ &\frac{\Gamma(K^- \to \pi^+ \mu^- \mu^-)}{\Gamma(K^- \to \mathrm{all})} < 1.5 \cdot 10^{-4} \\ &\quad \Rightarrow \langle m_{\mu\mu} \rangle < 1.3 \cdot 10^5 \text{ GeV}. \end{aligned} \quad (3)$$

Here the experimental limits are taken from the PDG [9] and for the mass bounds the theoretical results given in [10] and [7] are used (all ratios are proportional to $\langle m_{\mu\alpha} \rangle^2$). For muon–positron conversion two results are given, depending on whether the proton pairs in the final state are in a spin singlet or triplet state, respectively. To our knowledge, there are no direct limits on other elements of $\langle m_{\alpha\beta} \rangle$. Note that we are considering *direct* limits, i. e. measuring processes which are directly dependent on

[a] e-mail: flanz@dilbert.physik.uni-dortmund.de
[b] e-mail: rodejoha@dilbert.physik.uni-dortmund.de
[c] e-mail: zuber@physik.uni-dortmund.de

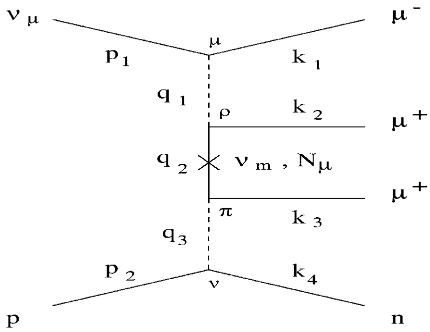

Fig. 1. Feynman diagram for the considered process. It is $q_2 = q_1 - k_2 = p_1 - k_1 - k_2$. For the crossed diagram k_2 and k_3 are exchanged and we denote the corresponding momentum of the Majorana neutrino with $\tilde{q}_2 = q_1 - k_3 = p_1 - k_1 - k_3$. For the W momenta holds: $q_1 = p_1 - k_1$ and $q_3 = k_4 - p_2$

the respective quantity without making any further assumptions. We will show that using the scattering process (2) instead of the rare K decay allows to set a limit better by one order of magnitude on $\langle m_{\mu\mu} \rangle$, which is however still too high to give a physical mass matrix in (1). *Indirect* bounds, e. g. using unitarity of the mixing matrix and oscillation experiments will of course be much more stringent. Direct production of Majorana neutrinos heavier than 100 GeV has been studied for various collider types (e^-e^+, $e\mu$, pp, $p\overline{p}$, e^-p) [11,12] with typical results of a few to some hundred events per year for high–energy and luminosity machines.

2 Model and calculation

Using the diagram shown in Fig. 1 plus its crossed version we get for the squared invariant amplitude three terms, each factorizing nicely in three parts. At the upper and lower vertex we have the standard V–A term. The contribution of the Majorana neutrino, i. e. the part $W^+W^+ \to \mu^+\mu^+$ is well known from the theory of $0\nu\beta\beta$. For the calculation one might follow the strategy in Kayser's textbook [13] or use the Feynman rules from [14], we will do the former. From here on we refer to this part of the diagram as the "$0\nu\beta\beta$–like" process. We use for particles the standard Lagrangian

$$\mathcal{L} = -\frac{g}{\sqrt{2}} \sum_m U_{\mu m} \overline{\mu} \gamma_\alpha \gamma_- U_{\mu m} \nu_m W^\alpha \tag{4}$$

where $\gamma_\pm = \frac{1}{2}(1 \pm \gamma_5)$. We denote the Majorana neutrino with ν_m, the muon with μ and $U_{\mu m}$ is an element of the unitary matrix connecting weak interaction eigenstates with mass eigenstates. For the $0\nu\beta\beta$–like contribution to the first matrix element we have:

$$\mathcal{M}_1 \propto \left[\overline{\nu_m} \gamma_\rho \gamma_+ U^*_{\mu m} \mu\right] \left[\overline{\nu_m} \gamma_\pi \gamma_+ U^*_{\mu m} \mu\right]. \tag{5}$$

To bring this in a form suitable for inserting the fermion propagator we use the relation:

$$\overline{\nu_m} \gamma_\rho \gamma_+ \mu = -\overline{\mu^c} \gamma_\rho \gamma_- \nu_m^c. \tag{6}$$

Here μ^c means the charge conjugated spinor of the muon. For a given spinor ψ charge conjugation has the properties:

$$\begin{aligned} \psi^c &= C\overline{\psi}^T, \quad \overline{\psi^c} = -\psi^T C^{-1} \\ C^{-1}\gamma_\mu C &= -\gamma_\mu^T, \quad C^{-1}\gamma_5 C = \gamma_5^T = \gamma_5. \end{aligned} \tag{7}$$

In the standard Dirac notation $C = i\gamma_2\gamma_0$ is the charge conjugation matrix. Since ν_m is a Majorana particle it has the property $\nu_m^c = \lambda_m^* \nu_m$, λ_m being a phase factor in the field expansion of ν_m connected with the intrinsic CP parity η_m^{CP}, see e. g. [13]. For the expansion in terms of spinors and creation/annihilation operators the following relations are valid:

$$\begin{aligned} \nu_m &\propto fu + \lambda_m f^+ v, \quad \mu \propto fu + \overline{f}^+ v, \\ \mu^c &\propto \overline{f}u + f^+ v, \quad \overline{\mu^c} \propto \overline{f}^+ \overline{u} + f\overline{v}. \end{aligned} \tag{8}$$

Here f annihilates a particle and $\overline{f}^+$ creates an antiparticle. Using all the above equations the matrix element describing the $0\nu\beta\beta$–like process can be written as (see Fig. 1 for the definition of the kinematics)

$$\begin{aligned} \mathcal{M}_1 &\propto \lambda_m^* U_{\mu m}^{*\,2} \overline{\mu^c} \gamma_\rho \gamma_- \nu_m \overline{\nu_m} \gamma_\pi \gamma_+ \mu \\ &= \lambda_m^* U_{\mu m}^{*\,2} \left[\overline{u}(k_2) \gamma_\rho \gamma_- (\not{q}_2 + m_m) \gamma_\pi \gamma_+ v(k_3)\right] \frac{1}{q_2^2 - m_m^2} \\ &= \lambda_m^* U_{\mu m}^{*\,2} m_m \left[\overline{u}(k_2) \gamma_\rho \gamma_- \gamma_\pi v(k_3)\right] \frac{1}{q_2^2 - m_m^2}. \end{aligned} \tag{9}$$

From here on we neglect the mass m_m in the denominator. See below for the case when this is no longer allowed. The above is the matrix element one would have obtained for an intermediate Dirac neutrino and applying the usual Feynman rules with one outgoing μ^+ written with an $\overline{u}$ instead of a v (thus producing a scalar expression) and one γ_+ replaced with a γ_-. Assuming CP invariance, the term $\lambda_m^* U_{\mu m}^{*\,2} m_m$ can be written as (see e. g. [13,15])

$$\left| \sum_m \lambda_m^* U_{\mu m}^{*\,2} m_m \right| = \left| \sum_m m_m \eta_m^{\mathrm{CP}} U_{\mu m}^2 \right| \equiv \langle m_{\mu\mu} \rangle, \tag{10}$$

thus defining the usual effective mass. The matrix element is therefore proportional to the effective Majorana mass, just as in $0\nu\beta\beta$ and the other mentioned lepton–number violating processes.

For the crossed diagram, described by $\mathcal{M}_2$, q_2 is replaced by $\tilde{q}_2$ and k_2 by k_3. Finally, the interference term is given by

$$-\mathcal{M}_1^* \mathcal{M}_2 \propto \overline{v}(k_3) \gamma_\nu \gamma_+ \gamma_\mu u(k_2) \overline{u}(k_3) \gamma_\alpha \gamma_- \gamma_\beta v(k_2) \tag{11}$$

which has a negative sign due to the interchange of two identical fermion lines. Using the identities $\overline{v} = -u^T C^{-1}$ and $u = C\overline{v}^T$ this can be written in a form suitable for using the completeness relations for the spinors:

$$\begin{aligned} \overline{v}(k_3) \gamma_\nu \gamma_+ \gamma_\mu u(k_2) &= -u^T(k_3) C^{-1} \gamma_\nu \gamma_+ \gamma_\mu C \overline{v}^T(k_2) \\ &= -u^T(k_3) \left(\gamma_\mu \gamma_+ \gamma_\nu\right)^T \overline{v}^T(k_2) \\ &= \overline{v}(k_2) \gamma_\mu \gamma_+ \gamma_\nu u(k_3). \end{aligned} \tag{12}$$

Putting all the couplings and propagators together, the matrix element for scattering with a quark can be written as (using the Feymann–gauge for the W propagator)

$$
\begin{aligned}
&|\overline{\mathcal{M}}|^2 \\
&= |\overline{\mathcal{M}_1}|^2 + |\overline{\mathcal{M}_2}|^2 + 2\Re\left(\overline{\mathcal{M}_1^*\mathcal{M}_2}\right) \\
&= |\langle m_{\mu\mu}\rangle|^2 64 G_F^4 M_W^8 \left|\frac{1}{q_1^2 - M_W^2}\frac{1}{q_3^2 - M_W^2}\right|^2 \\
&\quad \times \mathrm{Tr}\left\{\gamma^\mu\gamma_-\not{p}_1\gamma^\beta\gamma_-(\not{k}_1 + m_\mu)\right\} \\
&\quad \times \mathrm{Tr}\left\{\gamma^\nu\gamma_-(\not{p}_2 + m_q)\gamma^\alpha\gamma_-(\not{k}_4 + m_{q'})\right\} \\
&\quad \times \left[\left|\frac{1}{q_2^2}\right|^2 \mathrm{Tr}\left\{\gamma_\mu\gamma_-\gamma_\nu(\not{k}_3 - m_\mu)\gamma_\alpha\gamma_+\gamma_\beta(\not{k}_2 + m_\mu)\right\}\right. \\
&\quad + \left|\frac{1}{\tilde{q}_2^2}\right|^2 \mathrm{Tr}\left\{\gamma_\mu\gamma_-\gamma_\nu(\not{k}_2 - m_\mu)\gamma_\alpha\gamma_+\gamma_\beta(\not{k}_3 + m_\mu)\right\} \\
&\quad \left. -2\left|\frac{1}{q_2^2}\frac{1}{\tilde{q}_2^2}\right| \mathrm{Tr}\left\{\gamma_\beta\gamma_+\gamma_\alpha(\not{k}_3 + m_\mu)\gamma_\mu\gamma_-\gamma_\nu(\not{k}_2 - m_\mu)\right\}\right] \\
&= |\langle m_{\mu\mu}\rangle|^2 G_F^4 M_W^8 \left|\frac{1}{q_1^2 - M_W^2}\frac{1}{q_3^2 - M_W^2}\right|^2 2^{12}(p_1\cdot p_2) \\
&\quad \times \left[\left|\frac{1}{q_2^2}\right|^2 (k_1\cdot k_2)(k_3\cdot k_4) + \left|\frac{1}{\tilde{q}_2^2}\right|^2 (k_2\cdot k_4)(k_1\cdot k_3)\right. \\
&\quad - \left|\frac{1}{q_2^2}\frac{1}{\tilde{q}_2^2}\right| ((k_2\cdot k_3)(k_1\cdot k_4) - (k_2\cdot k_4)(k_1\cdot k_3) \\
&\quad \left. -(k_1\cdot k_2)(k_3\cdot k_4))\right]. \qquad (13)
\end{aligned}
$$

m_q and $m_{q'}$ are the masses of the scattered initial and final state partons, respectively. Coupling to an antiquark is identical to replacing k_4 with p_2. The two short traces describe the SM V–A vertices, the ones inside the square brackets are the $0\nu\beta\beta$–like process. Averaging over the parton spin adds an additional factor 1/2. The long traces were computed with the MATHEMATICA [16] package TRACER [17]. As can be seen, in the low mass regime the matrix element is proportional to $\langle m_{\mu\mu}\rangle^2$. If we take a heavy Majorana neutrino into account, one has to include the mass in the propagator for q_2 and $\tilde{q}_2$, which we neglected from (9) on. A statistical factor of 1/2 due to two identical final state muons has to be included in order to avoid double counting in the phase space integration.

We also performed the calculation for purely right–handed (RH) currents and obtained the exact same result with the exchange $W \to W_R$. As known, RH currents must occur — if they exist — strongly suppressed with respect to the left–handed ones. Since the W momenta are relatively small in comparison to M_W, the cross section is proportional to

$$\sigma \propto G_F^4 M_W^8((q_1^2 - M_W^2)(q_3^2 - M_W^2))^{-2} \sim G_F^4 \propto \left(\frac{g^2}{M_W^2}\right)^4, \qquad (14)$$

forcing the purely RH case to be some orders of magnitude under the purely left–handed case, since $M_{W_R} > 6M_W$ [9]. Here we assumed $g_L = g_R = g$.

One could also consider a heavy right–handed Majorana neutrino as suggested by some left–right symmetric theories [18], where leptons are arranged symmetrically in left–handed (LH) and RH doublets, i. e.

$$\begin{pmatrix}\nu_\mu \\ \mu^-\end{pmatrix}_{\mathrm{L}} \text{ and } \begin{pmatrix}N_\mu \\ \mu^-\end{pmatrix}_{\mathrm{R}}. \qquad (15)$$

Here N_μ is a heavy Majorana neutrino in a weak leptonic current of the form

$$j_l^\alpha = \overline{\mu}\gamma^\alpha\gamma_+ N_\mu + \overline{\mu}\gamma^\alpha\gamma_-\nu_\mu + \ldots + \text{h. c.} \qquad (16)$$

where the dots denote non muonic contributions. We consider it in order to illustrate the general properties of process (2) in a model independent way and to stress the fact that the greatest sensitivity is achieved for a Majorana mass of 1 to 10 GeV, independent of the exact form of the coupling to the W, see below. Furthermore it serves as a comparison to the results from [5,6], who also considered this possibility. In general all possibilities could contribute at the same time. Performing the same calculation as before we get for the N_μ– case in (13) a replacement ($\gamma_+ \leftrightarrow \gamma_-$) for the trace describing the $0\nu\beta\beta$–like process which leads in the end to a replacement ($k_1 \leftrightarrow p_1,\ k_4 \leftrightarrow p_2$). Again, antiquark scattering is obtained by replacing k_4 with p_2 in the quark amplitude.

Also possible is the exchange of other hypothetical particles such as those from the plethora of SUSY. Anyway, if process (2) could be detected, general arguments guarantee a Majorana mass term for the muon neutrino, just as the Schechter and Valle argument [19] does in the case for neutrinoless double beta decay for the electron neutrino. In [20] this theorem has been generalized to supersymmetry demanding also a non–vanishing Majorana mass for the scalar superpartners of the SM neutrinos. The smallness of the cross section however makes a more detailled analysis in this case not worthwhile: One could in principle derive limits on the right–handed coupling and/or W_R mass but they would definitely not compete with bounds derived by other methods, in contrast to the bound we will derive on $\langle m_{\mu\mu}\rangle$ in the next section.

3 Results and discussion

For the evaluation of the total and differential cross sections we wrote a Monte Carlo program calling the phase space routine GENBOD [21]. For the parton distributions we chose GRV 98 ($\overline{\mathrm{MS}}$) NLO [22] at $Q^2 = s = (p_1+p_2)^2 = x^2M_p^2 + 2xM_pE_\nu$, where M_p denotes the proton mass, E_ν the energy of the incoming neutrino and x the Bjørken variable. We set $Q^2 = Q^2_{\min}$ whenever Q^2 went under the minimal allowed value of $0.8\,\mathrm{GeV}^2$. To get the averaged neutrino–nucleon cross section we assumed an isoscalar target and replaced up– and down quarks to get the parton distributions for the neutron.

Before presenting the results we estimate the ratio with respect to the total neutrino–nucleon cross section. The typical suppression factor one encounters when dealing with Majorana instead of Dirac neutrinos is M/E in the matrix elements with M being the Majorana's mass and E its energy. For the ratio R of the cross sections we have therefore:

$$\mathrm{R} = \frac{\sigma(\nu_\mu \mathrm{N} \to \mu^-\mu^+\mu^+\mathrm{X})}{\sigma(\nu_\mu \mathrm{N} \to \mu^-\mathrm{X})} \propto \left(\frac{M}{E}\right)^2 G_F^2 M_W^4 \sim \begin{cases} 10^{-13} & \text{for } M = 170\text{ keV} \\ 10^{-5} & \text{for } M = 5\text{ GeV} \end{cases}, \quad (17)$$

where we took as a typical value $E = 30$ GeV. For heavy neutrinos the behavior changes significantly: Instead of M/E we have now M^{-2} (see below) and the ratio goes as

$$\mathrm{R} \propto \frac{G_F^2 M_W^4 M_p^2}{M^2} \sim 10^{-7} \text{ for } M = 100\text{ GeV} \quad (18)$$

These ratios will of course be further suppressed by a very small phase space factor, which rises slightly with energy and turns out to be about 10^{-7}, as well as by factors arising from bounds on the mixing with heavy neutrinos, see below.

As expected, the cross section is tiny: If we use for the mass of the Majorana neutrino the current limit from the direct muon neutrino mass search, $m_{\nu_\mu} = 170$ keV [23], we find that the cross section for energies in the range $5\ldots500$ GeV is of the order $\sigma(3\mu) \simeq 1\ldots10^2 \cdot 10^{-33}$b, being 20 orders of magnitude lower than the total neutrino nucleon CC cross section of $\sigma_{\mathrm{CC}} \simeq 1\ldots10^2 \cdot 10^{-14}$b for the same energy range. The E_ν dependence of the cross section can be fitted as a quadratically polynom, i. e. $\sigma(3\mu, E_\nu) = a \cdot E_\nu + b \cdot E_\nu^2$ which has to be compared to the linear dependence of the total CC neutrino–nucleon cross section. If we assume that this behavior holds up to ultrahigh energies (which it does not due to propagator effects) the cross sections would be roughly equal for $E_\nu \simeq 10^{20}$ GeV, far beyond any reasonable scale.

The scaling with $\langle m_{\mu\mu}\rangle^2$ holds up to masses of about 1 to 10 GeV. The RH N_μ produces a signal in the same order of magnitude. We plot the trimuon cross section in Fig. 2 together with the total CC neutrino nucleon cross section of about $0.7 \cdot 10^{-14}\, E_\nu/\mathrm{GeV}$ b, multiplied with 10^{-20}.

Despite the small values, the ratio of the trimuon process described here is significantly more sensitive on $\langle m_{\mu\mu}\rangle$ than other discussed processes: Abad et al. [7] get in a relativistic quark model for the decay $K^+ \to \pi^-\mu^+\mu^+$ a branching ratio of $2 \cdot 10^{-22}$ while Missimer et al. [4] estimate the ratio of $\mu^-\mu^+$– conversion via capture in ^{44}Ti with respect to a normal CC reaction to be $4 \cdot 10^{-25}$ for a few hundred keV Majorana. Thus the process (2) is about two orders of magnitude closer to the relevant standard model process than previously discussed Majorana induced muon–number violating processes.

Considering now the massive case, i. e. including the Majorana masses in the propagator, the squared amplitude will now be proportional to the sum

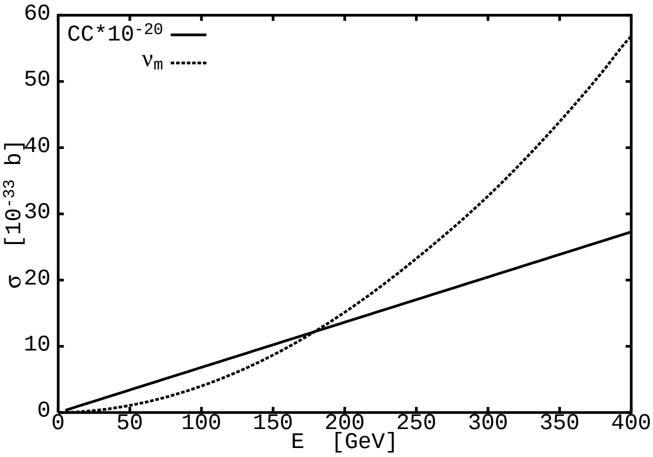

Fig. 2. Total cross section for process (2) with a left–handed (solid) ν_m together with the total CC νN cross section times 10^{-20} (dashed). The (effective) mass for the neutrinos is $\langle m_{\mu\mu}\rangle = 170$ keV

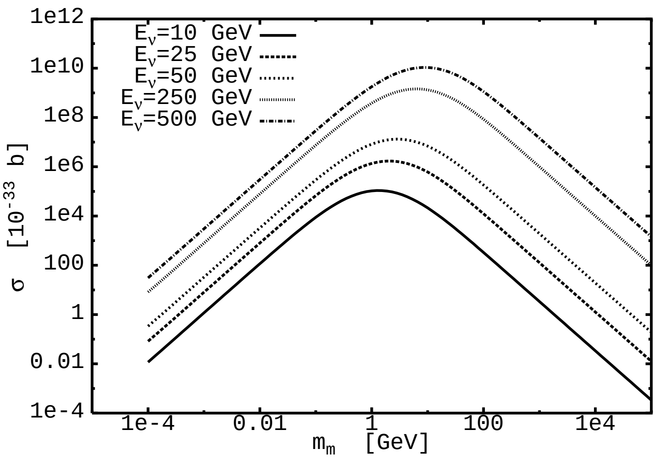

Fig. 3. Total cross section for a left–handed Majorana neutrino as a function of its mass for different neutrino beam energies. No limit on $U_{\mu m}^2$ was applied

$$\sigma \propto \left| \sum_m \frac{m_m \eta_m^{\mathrm{CP}} U_{\mu m}^2}{(q_2^2 - m_m^2)} \right|^2 . \quad (19)$$

For the sake of simplicity we skip for the moment the factors $\eta_m^{\mathrm{CP}} U_{\mu m}^2$ and consider only one mass eigenvalue, which turns out to dominate the cross section when it has an appropriate value. First of all, the cross section as a function of mass will rise quadratically until the propagator takes over and forces a $(\mathrm{mass})^{-2}$ behavior. This is displayed in Fig. 3 where we plot the total cross section for different neutrino energies. As can be seen the maximal value of the cross section as a function of mass is obtained in the range $1\ldots10$ GeV, rising slightly with E_ν. The reason for that is that the integration over the neutrino propagator has its maximum in this range. This fact makes the greatest sensitivity independent of the coupling of the Majoranas to the leptons or W's. One can show that the heavy right–

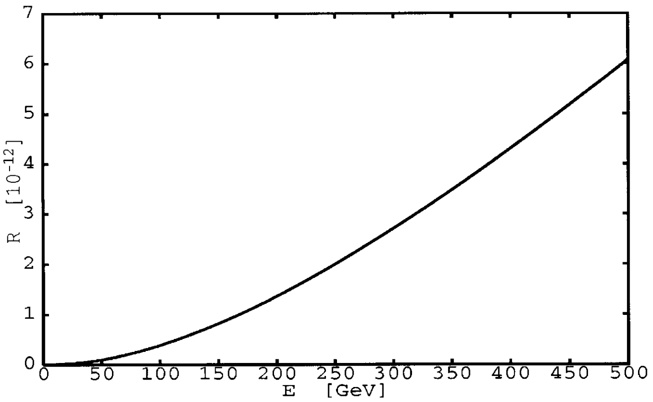

Fig. 4. Ratio for process (2) with respect to the total CC νN cross section for a left–handed ν_m of (effective) mass 5 GeV. No limit on $U^2_{\mu m}$ was applied

Table 1. Pure (R) and "$U_{\mu m}$–corrected" ($\mathrm{R_{cor}}$) ratios of the process for masses of 7 and 80 GeV and different neutrino beam energies in GeV

	$m_m = 7\,\mathrm{GeV}$		$m_m = 80\,\mathrm{GeV}$	
E_ν	R	$\mathrm{R_{cor}}$	R	$\mathrm{R_{cor}}$
25	$1.1 \cdot 10^{-14}$	$4.4 \cdot 10^{-24}$	$1.1 \cdot 10^{-16}$	$4.1 \cdot 10^{-21}$
50	$6.2 \cdot 10^{-14}$	$2.5 \cdot 10^{-23}$	$8.5 \cdot 10^{-16}$	$3.1 \cdot 10^{-20}$
100	$3.1 \cdot 10^{-13}$	$1.2 \cdot 10^{-22}$	$6.3 \cdot 10^{-15}$	$2.3 \cdot 10^{-19}$
250	$2.1 \cdot 10^{-12}$	$8.4 \cdot 10^{-22}$	$8.7 \cdot 10^{-14}$	$3.2 \cdot 10^{-18}$
500	$7.4 \cdot 10^{-11}$	$3.0 \cdot 10^{-20}$	$6.1 \cdot 10^{-13}$	$2.2 \cdot 10^{-17}$

handed N_μ displays the same behavior as the left–handed Majorana case shown in Fig. 3, which underlines this fact. In Fig. 4 we display the ratio with respect to the total CC neutrino–nucleon cross section for a mass of 5 GeV as a function of the incoming neutrino energy. Note that this light masses are ruled out [9] and that for higher masses ($M \gtrsim 100\,\mathrm{GeV}$) the ratio scales with M^{-2}.

Up to now all the numbers given were for $U^2_{\mu m} = 1$. In this case, a maximum of $7.4 \cdot 10^{-11}$ of the CC cross section would be reached for a Majorana with mass of about 7 GeV. A neutrino beam of 500 GeV, coming from a high energy and luminous $\mu^+\mu^-$– collider with 10^{13} CC events per year could in principle produce a few hundred of such events.

However, there exist already strong constraints on the matrix elements $U_{\mu m}$ from the data. The DELPHI collaboration [24] examined the mode $Z \to \overline{\nu}\nu_m$ and found a limit of $|U_{\mu m}|^2 < 2 \cdot 10^{-5}$ for masses up to $m_m \simeq 80\,\mathrm{GeV}$. For larger masses analyses of neutrino–quark scattering and other processes yield $|U_{\mu m}|^2 < 0.0060$ [25]. This pushes the best sensitivity range about a factor of 10 towards higher values of m_m.

In Table 1 we show the ratios R with and without taking into account the limits given above for different energies and for Majorana masses of 7 and 80 GeV. As can be seen one cannot get closer than at most 10^{-17}, even for a 500 GeV neutrino beam. In [12] finite width effects were found to increase the cross sections for direct heavy Majorana production significantly. However, these effects show up for high center–of–mass energies and high masses so that in our kinematical and mass sensitivity region these effects should be negligible.

Nevertheless, also in the massive case the improvement compared to existing numbers is some orders of magnitude: Halprin et al. [5] find a ($U_{\mu m}$–corrected) BR smaller than $3 \cdot 10^{-27}$ for $K^+ \to \pi^-\mu^+\mu^+$ and $\Sigma^+ \to \pi^-\mu^+\mu^+$ for a universally coupled 5 GeV heavy neutrino and Ng and Kamal [6] get a ($U_{\mu m}$–corrected) branching ratio of $1.3 \cdot 10^{-25}$ for a 2 GeV right–handed Majorana coupling to the W as in (16). This means, for $E_\nu = 100$ (500) GeV and few GeV Majoranas, process (2) is up to 5 (7) orders of magnitude closer to the standard model CC process than previously discussed muon–number violating $\Delta L_\mu = 2$ processes, which are induced by Majoranas. Interestingly the highest BR for the mentioned K decay in [5] is also in the range of 1 to 10 GeV.

Though the cross section is probably too small to detect this process in the near future, it still allows to set bounds on $\langle m_{\mu\mu}\rangle$. Let us assume an upper limit on a process like (2) of the order 10^{-5} of the standard CC process (otherwise it would have been observed already, see Sect. 4) and take an energy of $E_\nu = 100\,\mathrm{GeV}$. Starting at small masses, i. e. $\sigma \propto \langle m_{\mu\mu}\rangle^2$, we find $\langle m_{\mu\mu}\rangle \lesssim 10^4\,\mathrm{GeV}$. This has to be compared to $\langle m_{\mu\mu}\rangle < 1.3 \cdot 10^5\,\mathrm{GeV}$ as obtained from K–decays [26].

What is now the significance of this $\langle m_{\mu\mu}\rangle$ bound? Obviously, in a three–neutrino framework all elements of $\langle m_{\alpha\beta}\rangle$ should be roughly in the same order of magnitude, therefore at most a few eV, not much higher than the limit for $\langle m_{ee}\rangle$. In scenarios with additional massive neutrinos one has to include other processes as restrictions for possible mass matrix models. The bounds for mixing matrix elements with heavy neutrinos are typically in the order of a few 10^{-3} to 10^{-2} for ν_e, ν_μ and ν_τ, thus roughly the same for all three families. It is impossible to reconcile the bounds for $\langle m_{ee}\rangle$, $\langle m_{e\mu}\rangle$ and $\langle m_{\mu\mu}\rangle$ with these conditions. If one considers heavy neutrinos and allows $\langle m_{ee}\rangle$ to be as high as the other two one can in principle fulfill the other conditions, but one is lead to contradictions when taking flavor changing neutral current processes into account, see the Appendix. It is thus not possible to construct a mass matrix $\langle m_{\alpha\beta}\rangle$ spanning 14 orders of magnitude from $\langle m_{ee}\rangle$ to $\langle m_{\mu\mu}\rangle$. Therefore, indirect bounds will be far more effective than direct ones.

4 Experimental considerations

Several experiments report the observation of trimuon events [27–29]. The observed ratio of trimuon events (having a lepton number conserving ($- - +$) signature) with respect to single charged current events is of the order 10^{-5}. First thought to provide evidence for physics beyond the SM the explanation was soon given in terms of CC reactions with dimuon production via meson decay, radiative processes or direct muon pair production from subsequent hadronic interactions [30–32]. An acceptance

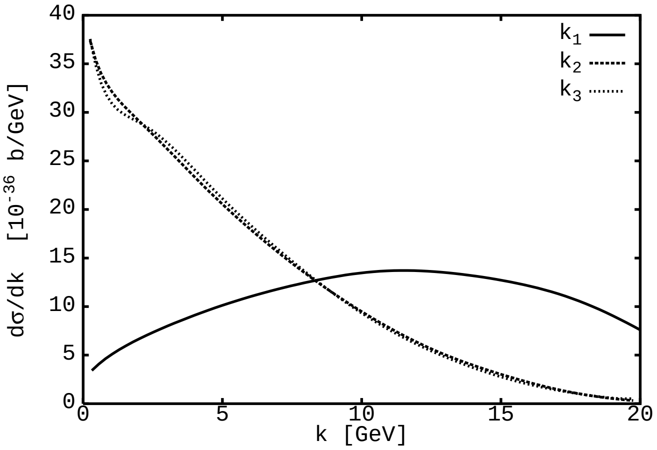

Fig. 5. Differential cross section at $E_\nu = 25\,\mathrm{GeV}$ for the momenta of the three muons for the case of a left–handed Majorana with effective mass of 170 keV. k_1 is the muon momentum from the standard V–A vertex for the incoming neutrino, k_2 and k_3 are the muons from the $0\nu\beta\beta$–like vertices

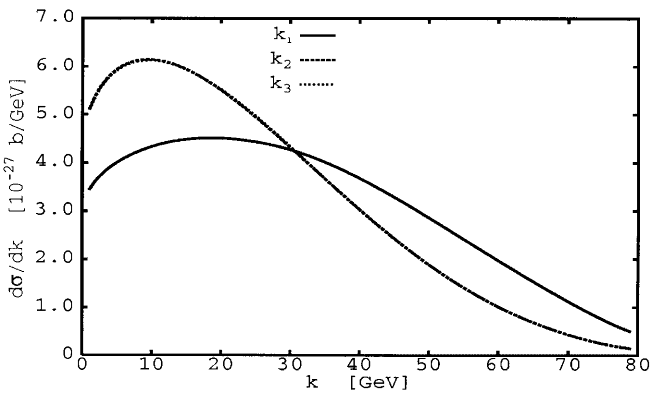

Fig. 6. Same as previous figure for a mass of 5 GeV and incoming neutrino energy of 100 GeV. No limit on $U^2_{\mu m}$ was applied

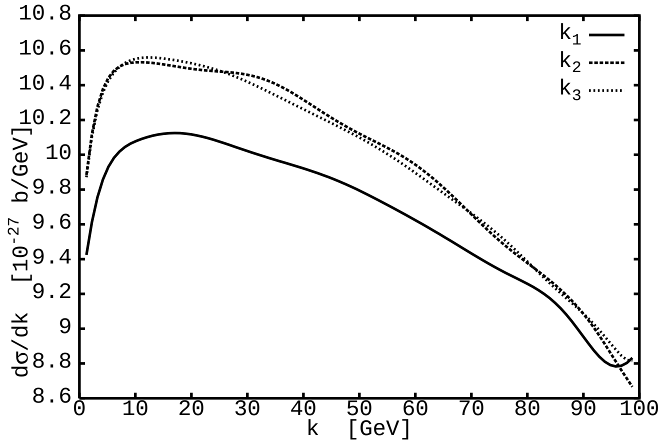

Fig. 7. Same as above for a mass of 80 GeV and incoming neutrino energy of 500 GeV. Note that in this case the momenta of the two μ^+ are always larger than the momentum of the μ^-. No limit on $U^2_{\mu m}$ was applied

cut on muon momenta to be larger than about 5 GeV was applied by all experiments.

To extract a $(- + +)$ signature several background processes in typical wide band neutrino beams have to be considered. Among them the most severe are lepton pair creation due to antineutrino contaminations of the beam (also having a $(- + +)$ signature) and charm production with an associated pion or kaon decay as well as overlaying events with beam muons. Furthermore, going to high momenta some misidentification in the charge might lead to additional background.

The observables found in the past to be most suitable for distinguishing the mentioned standard processes from new physics were the momenta of the muons, their two– and three–body invariant masses and the azimuthal angle distribution between the leading muon and the other two. The leading muon (μ_1) was defined as the one which minimizes the sum of the transverse momenta of the remaining two with respect to the direction of the W, ($\vec{W} = \vec{\nu} - \vec{\mu}_1$). A complete listing of all relevant distributions is not our aim, however, for the sake of completeness, we plot the distributions of the muon momenta, which might be used to identify the process. From Figs. 5 to 7 it can be seen, that for the light $\langle m_{\mu\mu}\rangle$– case the two μ^+ have relatively low energy, while the μ^- from the V–A vertex has a broad spectrum with significantly higher energy. This is no longer valid for a heavy Majorana where the difference of the muon momenta is less clear, but is becoming larger with increasing neutrino energy. However, the like–sign muons have typically the same momentum distributions, which is an important experimental signature. It is a general feature that the momentum difference gets bigger when the energy E_ν is significantly higher than the mass of the intermediate Majorana. For mass and energy being equal the distributions are more or less identical.

A similar search to the one described here could also be done with $\bar{\nu}_\mu$ beams looking for the corresponding process $\bar{\nu}_\mu \mathrm{N} \to \mu^+\mu^-\mu^- X$, which though have typically a lower cross section of one order of magnitude.

5 Summary and conclusion

We investigated the reaction $\nu_\mu N \to \mu^-\mu^+\mu^+ X$ at fixed target experiments mediated by light and heavy Majorana neutrinos. Using the fact, that no excess events were observed in past experiments at the level of 10^{-5} with respect to charged current events, we could deduce a limit of $\langle m_{\mu\mu}\rangle \lesssim 10^4$ GeV. This is more stringent than other *direct* results discussed on this quantity, but obviously not reconcilable with other laboratory experiments.

Some general properties of process (2) were discussed: The largest sensitivity was found for heavy Majorana neutrinos in the region between 1 and 10 GeV because of the fixed target kinematics. This was pushed towards approximately 100 GeV due to existing limits on $U^2_{\mu m}$. This is relatively independent of incoming neutrino energy and independent on the precise form of the couplings, as can be shown with a right–handed Majorana. In general, process (2) is closer to the standard model CC process by 2 (up to a few 100 keV mass) up to 7 (> 1 GeV mass) orders of magnitude than previously discussed Majorana induced $\Delta L_\mu = 2$ processes. We state again that our 10^4 GeV is the best *direct* limit, not the best achievable limit.

One could consider various modifications of process (2) in order to constrain non–standard model parameters connected with the muonic sector. For the case of $0\nu\beta\beta$ limits on some Yukawa couplings $\lambda^{(')}_{1jk}$, describing R–Parity violating SUSY effects were deduced [33], the bounds being up to four orders of magnitude more stringent than the ones obtained from other processes. In addition, for muon capture in ^{44}Ti, extensions of the standard model were found [4] to have branching ratios some orders of magnitude higher than the Majorana case, so that it seems worthwhile to apply them to process (2) as well.

The smallness of the cross section however makes such a detailled analysis not very worthwhile. This might change for the case of a neutrino factory from a muon storage ring with a large number of interactions. Work about this topic is in progress and will be presented in the near future.

We concentrated our analysis on neutrino beams, especially ν_μ. Since the beam energies are much higher than the lepton masses, the same arguments as described here would hold for other fixed–target experiments using charged lepton beams. However, new background processes have to be considered here.

Furthermore, also a lepton–hadron collider such as HERA, which also has the advantage of higher $\sqrt{s}$ can be used. The same strategy that lead to the bound on $\langle m_{\mu\mu}\rangle$ can of course be applied to infer quantities as $\langle m_{\mu\tau}\rangle$ or $\langle m_{\tau\tau}\rangle$, for which no direct limits whatsoever exist. Taking the appropriate channels, no SM processes faking the signal exist. This has in the meantime been discussed in detail in [34].

Acknowledgements. This work has been supported in part (M. F. and W. R.) by the “Bundesministerium für Bildung, Wissenschaft, Forschung und Technologie, Bonn under contract number 05HT9PEA5. A scholarship (W. R.) of the Graduate College “Erzeugung und Zerfälle von Elementarteilchen” at Dortmund university is gratefully acknowledged.

A Appendix

The task is to construct a mass matrix with $\langle m_{ee}\rangle \simeq 0$, $\langle m_{e\mu}\rangle \simeq 2$ GeV and $\langle m_{\mu\mu}\rangle \simeq 10^4$ GeV. For the matrix elements for mixing with heavy neutrinos holds [25]

$$\sum |U_{eH}|^2 < 6.6 \cdot 10^{-3} \text{ and} \sum |U_{\mu H}|^2 < 6.0 \cdot 10^{-3}. \quad (20)$$

First, let us assume that $U^2_{eH} \simeq 0$ and $U^2_{\mu H} = 6 \cdot 10^{-3}$. Then we find that $m_H = 1.7 \cdot 10^6$ GeV, leading to $U_{eH} \simeq 1.6 \cdot 10^{-5}$ which in turn leads to a contribution to $\langle m_{ee}\rangle$ of $4.1 \cdot 10^5$ eV, in contradiction to our assumption. In general we found no solution for the allowed parameters.

Conversely, we might want to use the fact that there is a bound on heavy ($m_H > 1$ GeV) contributions from the Heidelberg–Moscow experiment of [35]

$$\sum U^2_{eH_i} \frac{1}{m_{H_i}} < 5 \cdot 10^{-5} \text{TeV}^{-1}. \quad (21)$$

Ignoring the condition $\langle m_{ee}\rangle \simeq 0$ allows to find parameters capable of obeying the $\langle m_{e\mu}\rangle$ and $\langle m_{\mu\mu}\rangle$ limits as well as Eqs. (20) and (21). For example, $U^2_{eH_1} = U^2_{eH_2} = 10^{-10}$, $U^2_{eH_3} = 4 \cdot 10^{-13}$ and $U^2_{\mu H_1} = 5 \cdot 10^{-3}$, $U^2_{\mu H_2} = -5 \cdot 10^{-4}$, $U^2_{eH_3} = 10^{-5}$ with $m_{H_1} = 100$ GeV, $m_{H_2} = 1$ TeV and $m_{H_3} = 10^6$ TeV.

Then again we have FCNC processes like $\mu \to e\gamma$, which are sensitive on $\overline{m_{e\mu}} = \sqrt{\sum U_{\mu i} U_{ei} m_i^2}$. The experimental value of the branching ratio, BR $< 1.2 \cdot 10^{-11}$ [36] and the theoretical value from [18] gives $\overline{m_{e\mu}} < 1.23$ GeV, which is not fulfilled by the choice given.

References

1. K. Zuber, Phys. Rep. **305**, 295 (1998)
2. For a review see: R. N. Mohapatra, Unification and Supersymmetry, 2nd edition, Springer Verlag, 1992; P. Langacker Phys. Rep. **72**, 185 (1981)
3. L. Baudis et al., Phys. Rev. Lett. **83**, 411 (1999)
4. J. H. Missimer, R. N. Mohapatra, N. C. Mukhopadhyay, Phys. Rev. D **50**, 2067 (1994)
5. A. Halprin, P. Minkowski, H. Primakoff, S. P. Rosen, Phys. Rev. D **13**, 2567 (1976)
6. J. N. Ng, A. N. Kamal, Phys. Rev. D **18**, 3412 (1978)
7. J. Abad, J. G. Esteve, A. F. Pachero, Phys. Rev. D **30**, 1488 (1984)
8. L. S. Littenberg, R. E. Shrock, Phys. Rev. Lett. **68**, 443 (1992)
9. Review of Particle Properties, C. Caso et al., Eur. Phys. J. C **3**, 1 (1998)
10. M. Doi, T. Kotani, E. Takasugi, Prog. Theor. Phys. Suppl. **83**, 1 (1985)
11. S. T. Petcov, Phys. Lett. B **139**, 421 (1984); F. del Aguila, E. Laerman, P. Zerwas, Nucl. Phys. B **297**, 1 (1988); W. Buchmüller, C. Greub, Nucl. Phys. B **363**, 345 (1988); E. Ma, J. Pantaleone, Phys. Rev. D **40**, 2172 (1989); J. Kogo, S. Y. Tsai, Prog. Theor. Phys. **86**, 183 (1991); D. A. Dicus, D. D. Karatas, P. Roy, Phys. Rev. D **44**, 2033 (1991); J. Maalampi, K. Mursula, R. Vuopionperä, Nucl. Phys. B **372**, 23 (1992); J. Gluza, M. Zralek, Phys. Rev. D **48**, 5093 (1993); A. Datta, M. Guchait, D. P. Roy Phys. Rev. D **47**, 961 (1993); A. Datta, A. Pilaftsis, Phys. Rev. D **50**, 3195 (1994); A. Hoefer, L. M. Sehgal, Phys. Rev. D **54**, 1944 (1996); J. Gluza, M. Zralek, Phys. Rev. D **55**, 7030 (1997); F. M. L. Almeida et al., Phys. Lett. B **400**, 331 (1997); P. Panella, C. Carimalo, Y. N. Srivastava, hep-ph/9903253
12. G. Cvetic, C. S. Kim, C. W. Kim, Phys. Rev. Lett. **82**, 4761 (1999), G. Cvetic, C. S. Kim, Phys. Lett. B **461**, 248 (1999)
13. B. Kayser, F. Gibrat–Debu, F. Perrier, The Physics of Massive Neutrinos, World Scientific, 1989
14. H. E. Haber, G. L. Kane, Phys. Rev. **117**, 75 (1985)

15. S. M. Bilenky, S. T. Petcov, Rev. Mod. Phys. **59**, 671 (1987)
16. S. Wolfram, Mathematica, Addison–Wesley, 1991
17. M. Jamin, M. E. Lautenbacher, Comp. Phys. Comm. **74**, 265 (1993)
18. J. C. Pati, A. Salam, Phys. Rev. D **10**, 275 (1974); R. N. Mohapatra, J. C. Pati Phys. Rev. D **11**, 566, 2558 (1975); G. Senjanovic, R. N. Mohapatra, Phys. Rev.D **12**, 1502 (1975), R. N. Mohapatra, P. B. Pal, Massive Neutrinos in Physics and Astrophysics, 2nd edition, World Scientific, Singapore, 1998
19. J. Schechter, J. W. F. Valle, Phys. Rev. D **25**, 2951 (1981)
20. M. Hirsch, H. V. Klapdor-Kleingrothaus, S. G. Kovalenko, Phys. Lett. B **398**, 311 (1997), ibid B **403**, 291 (1997)
21. F. James, CERN 68–15 (1968)
22. M. Glück, E. Reya, A. Vogt, Eur. Phys. J. C **5**, 461 (1998)
23. K. Assamagan et al., Phys. Rev. D **53**, 6065 (1996)
24. P. Abreu, Z. Phys. C **74**, 57 (1997), erratum ibid. C **75** 580 (1997)
25. E. Nardi, E. Roulet, D. Tommasini, Phys. Lett. B **344**, 225 (1995)
26. H. Nishiura, K. Matsuda, T. Fukuyama, Mod. Phys. Lett. A **14**, 433 (1999)
27. B. C. Barish et al., Phys. Rev. Lett. **38**, 577 (1977)
28. A. Benvenuti et al., Phys. Rev. Lett. **38**, 1110 (1977), Phys. Rev. Lett. **38**, 1183 (1977), Phys. Rev. Lett. **42**, 1024 (1979)
29. M. Holder et al., Phys. Lett. B **70**, 393 (1977), T. Hansl et al., Nucl. Phys. B **142**, 381 (1978)
30. V. Barger, T. Gottschalk, R. J. N. Phillips, Phys. Rev. D **17**, 2284 (1978), Phys. Rev. D **18**, 2308 (1978)
31. J. Smith, J. A. M. Vermasseren, Phys. Rev. D **17**, 2288 (1978)
32. R. M. Barnett, L. N. Chang, N. Weiss, Phys. Rev. D **17**, 2266 (1978)
33. R. N. Mohapatra, Phys. Rev. D **34**, 3457 (1986); M. Hirsch, H. V. Klapdor–Kleingrothaus, S. Kovalenko, Phys. Rev. Lett. **75**, 17 (1995), Phys. Lett. B **352**, 1 (1995); K. S. Babu, R. N. Mohapatra, Phys. Rev. Lett. **75**, 2276 (1995); M. Hirsch, H. V. Klapdor–Kleingrothaus, S. Kovalenko, H. Päs, Phys. Rev. D **53**, 1329 (1996); H. V. Klapdor–Kleingrothaus, Proc. NEUTRINO 96, Helsinki, June 1996
34. M. Flanz, W. Rodejohann, K. Zuber, Phys. Lett. B **473**, 324 (2000)
35. G. Belanger et al., Phys. Rev. D **53**, 6292 (1996)
36. M. L. Brooks et al. (MEGA collaboration), Phys. Rev. Lett. **83**, 1521 (1999)